A composition method approach for retinal imaging enhancement

Francesco Ballerin

francesco.ballerin@uib.no

Joint work with Erlend Grong

Abstract

In mammals **image inpainting** is intrinsically performed at the level of the **Primary Visual Cortex V1** in order to fill the gaps in the field of view and make sense of the surroundings when obstructions are present. Petitot and Tondut [1] have proposed to model this part of the brain as $SE(2)$, a **sub-Riemannian manifold**, where the neural activity is propagated through **anisotropic diffusion**. Solving the diffusion equation yields image restoration.

Milestones of this approach are the papers from Citti and Sarti [2] and Boscain et al. [3].

The aim of this work is to propose a new biologically inspired lift operator (**Gaussian lift**) from $\mathbb{R}^2$ to $PT\mathbb{R}^2$ and a versatile integrator based on composition methods (**WaxOn-WaxOff**) for image restoration and enhancement.

$SE(2)$ as a model for the Primary Visual Cortex V1

Neurons are grouped into **simple cells** that are sensitive to intensity and orientation at a specific position on the retina. Simple cells sensitive to a specific position, regardless of orientations, are grouped into **hypercolumns**. Excitatory synapses are present between simple cells with similar orientations located in the same hypercolumn and simple cells with same orientation located in spatially close hypercolumns. Synapses in the same hypercolumn are said to be **vertical** while synapses between simple cells in different hypercolumns with same orientation are said to be **horizontal**.

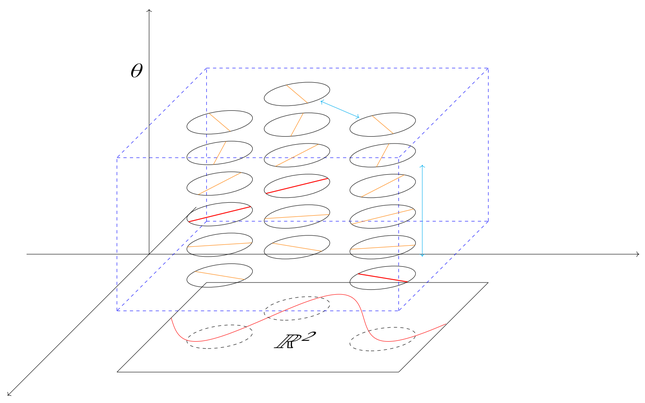


Figure: Visualization of the hypercolumns of the Visual Cortex V1 under a stimulus. The red orientation columns receive direct stimulus from the image while the orange ones through excitatory and inhibitory synapses. In cyan are visualized two directions of the excitatory synapses, inside a common hypercolumn and between hypercolumns that are close spatially.

Such space is naturally modeled as $SE(2)$ as in the following definition:

$$SE(2) = \left\{ \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix} \middle| \; x, y \in \mathbb{R},\ \theta \in [0, 2\pi) \right\}$$

$$SE(2) \simeq \mathbb{R}^2 \times S^1$$

Define the following basis of left invariant vector fields by left translation:

$$\begin{aligned} \partial_x|_{\mathbf{I}} &\rightarrow X_1 = \cos\theta\,\partial_x + \sin\theta\,\partial_y \\ \partial_\theta|_{\mathbf{I}} &\rightarrow X_2 = \partial_\theta \\ \partial_y|_{\mathbf{I}} &\rightarrow X_3 = -\sin\theta\,\partial_x + \cos\theta\,\partial_y \end{aligned}$$

$SE(2)$ equipped with $\mathcal{H} = \{X_1, X_2\}$ as horizontal distribution and a metric g that makes X_1 and X_2 orthonormal (i.e. if $a = a_1X_1 + a_2X_2$ then $g(a,a) = \sqrt{a_1^2 + a_2^2}$) turns into a sub-Riemannian geometry. X_1 corresponds to the direction of the horizontal synapses while X_2 to the direction of vertical synapses.

The Primary Visual Cortex V1 can also be modeled as

$$PT\mathbb{R}^2 := \mathbb{R}^2 \times P^1$$

The geometric properties and sub-Riemannian structure on $PT\mathbb{R}^2$ are analogous to the ones on $SE(2)$.

The usual restoration algorithm

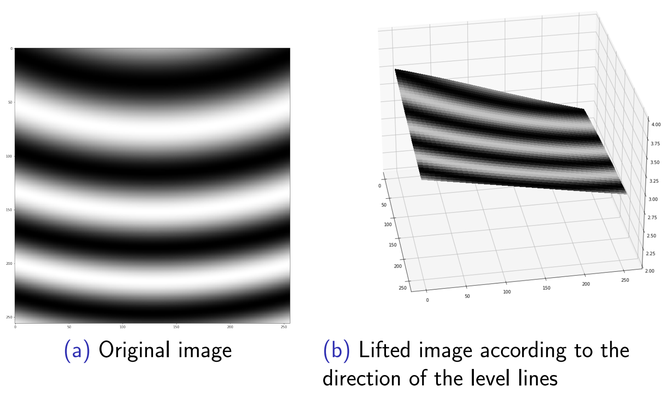
The algorithm is **blind** as it is not known which pixels are corrupted and which are not. Image restoration is achieved by the following 4 steps, starting with a grayscale digital image.

1. **Smooth** the image with a Gaussian kernel (guarantees the image is generically a Morse function)
2. **Lift** $\mathbb{R}^2 \rightarrow SE(2)$
3. Apply **diffusion** in the directions of X_1 and X_2 over $SE(2)$
4. **Project** $SE(2) \rightarrow \mathbb{R}^2$

The Citti and Sarti lift $\mathbb{R}^2 \rightarrow SE(2)$

For a Morse function lift the image to the surface in $SE(2)$ such that all the points are lifted corresponding to the orientation of the **level lines** at that point.

$$\tilde{I}(x,y,\theta) := \begin{cases} I(x,y) & |X_3(\theta)| = \max_{\bar{\theta}} |X_3(\bar{\theta})| \\ 0 & \text{otherwise} \end{cases}$$



(a) Original image

(b) Lifted image according to the direction of the level lines

A left-inverse to such lift is the projection of the maximum value along each fiber.

Diffusion equation on $SE(2)$

$$\begin{cases} \partial_t u = \Delta_H u \\ u(0,x,y,\theta) = \mathcal{L}(I(x,y,\theta)) \end{cases}$$
$$\Delta_H = X_1^2 + \beta^2 X_2^2$$

where β expresses the strength of the vertical synapses with respect to the horizontal ones.

Gaussian lift

From research in biology (Marcelja [4], Jones and Palmers [5]) we know that simple cells act like **Gabor filters**. In particular the activity level **decays exponentially** the more the orientation of the simple cells differ from the orientation perceived.

A **new lift operator** that follows such assumption is defined as

$$\mathcal{L}(I)(x,y,\theta) = I(x,y) \cdot \exp\left(-\frac{\left\langle \frac{\nabla I(x,y)}{|\nabla I(x,y)|}, (\cos\theta, \sin\theta) \right\rangle^2}{2\sigma^2} \right)$$

with left-inverse

$$\pi(\tilde{I})(x,y) = \frac{1}{\pi} \int_0^\pi \tilde{I}(x,y,\theta) \cdot e^{-\mu(x,y,\theta)} d\theta$$
$$\mu(x,y,\theta) := \min_{\tilde{\theta} \in [0,\pi]} Q(x,y,\tilde{\theta}) - Q(x,y,\theta)$$
$$Q(x,y,\theta) := -\int_0^\theta \frac{\partial}{\partial\tilde{\theta}} \ln(\tilde{I}(x,y,\tilde{\theta}))\, d\tilde{\theta}$$

Empirical results show that this lifting operator achieves better results for larger values of σ. A computationally less expensive projection which yields comparable results is the projection of the maximum value along each fiber.

WaxOn - WaxOff

"Wax on, left hand. Wax off, right hand. Wax on, wax off. [...] Don't forget to breath....very important!"

(Mr. Miyagi, The Karate Kid)

It is possible to model the numerical integrator for the diffusion equation as a composition method

$$\Psi_{T_1,T_2} = \mathcal{L} \circ \pi \circ \Phi_{-T_2} \circ \mathcal{L} \circ \pi \circ \Phi_{T_1}$$

where Φ is a simple numerical integrator (for example Euler's method) or alternatively

$$\Psi_{T_1,T_2} = \mathcal{L} \circ \pi \circ \Phi^*_{-T_2} \circ \mathcal{L} \circ \pi \circ \Phi_{T_1}$$

where Φ^* is the adjoint of Φ. An example in the case of image inpainting is the following:

(a) Original image

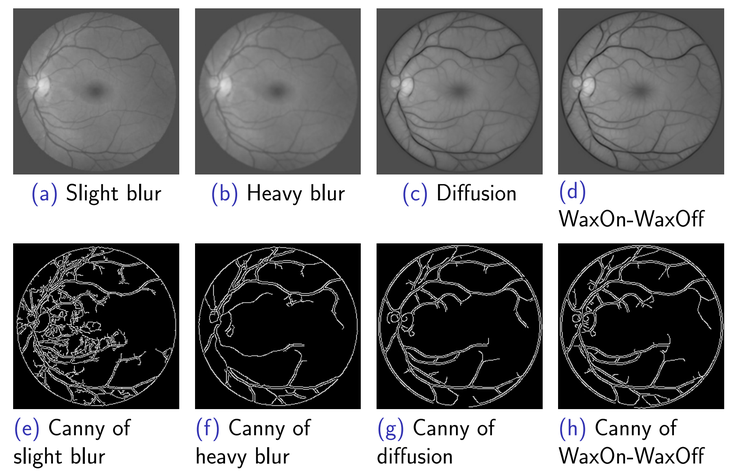
(b) Usual diffusion

(c) 1 step WaxOn-WaxOff

(d) 3 steps WaxOn-WaxOff

Figure: The original image (a) is lifted with Gaussian lift. The usual algorithm (our implementation) is applied to obtain (b). One iteration of *WaxOn-WaxOff* for small T_2 produces (c) while multiple iterations of WaxOn-WaxOff are sequentially applied to produce (d). The total diffusion time in (b) and (d) is the same.

An application with contour enhancement as the objective is the preprocessing of retinal images, for example prior to applying edge detection filters.



(a) Slight blur

(b) Heavy blur

(c) Diffusion

(d) WaxOn-WaxOff

(e) Canny of slight blur

(f) Canny of heavy blur

(g) Canny of diffusion

(h) Canny of WaxOn-WaxOff

Figure: Example of application of WaxOn-WaxOff to a retinal image (courtesy of Mikael Häggström, used with permission) as a preprocessing step prior to Canny edge detection. With the proposed method more high-frequency information is preserved.

By applying the two steps **repeatedly** the WaxOn portion of the algorithm yields restoration while the WaxOff portion sharpens the result while retaining restored information. Together with the Gaussian lift the resulting method appears promising.

References

[1]: J. Petitot and Y. Tondut, "Vers une neurogéométrie. Fibrations corticales, structures de contact et contours subjectifs modaux" Math. Inform. Sci. Humaines, 145, 5–101 (1999)
[2]: G. Citti and A. Sarti. "A cortical based model of perceptual completion in the roto-translation space", J. Math. Imaging Vis. (2006)
[3]: U. Boscain et al. "Anthropomorphic image inpainting through hypoelliptic diffudion", JSIAM j. control optim. (2012)
[4]: S. Marcelja. "Mathematical description of the responses of simple cortical cells". en. In: J. Opt. Soc. Am. (1980)
[5]: J. P. Jones and L. A. Palmer. "An evaluation of the two-dimensional Gabor filter model of simple receptive fields in cat striate cortex". en. In: J. Neurophysiol. (1987)

Research supported by the generous contribution of:

TROND MOHN STIFTELSE

L. MELTZERS HØYSKOLEFOND